Week 9 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
   `23f3004032@ds.study.iitm.ac.in`
   or reach out on Discourse: `@23f3004032`

Quiz 1 May 22

Quiz 2 May 22

ET May 22

1. A manufacturing company makes two types of television sets; smart led (x) and normal led (y). The objective function to maximise the profit, given profits of Rs 510 per smart led and Rs 675 per normal led, is:

   (a) $510x + 675y$

   (b) $410x + 675y$

   (c) $510x + 575y$

   (d) $102x + 135y$

   [Ans: 1]

2. For the television manufacturing problem, with a limit of 300 sets total and a budget of Rs 648,000 (Costs: Rs 1800/smart, Rs 2700/normal), the constraints are:

   (a) $x + y \leq 300$

   (b) $2x + 3y \leq 720$

   (c) $x + 3y \leq 120$

   (d) $x \geq 0, y \geq 0$

   [Ans: 1, 2, 4]

3. Data is generated by $Y = X + 5$. A learned model is $f(x) = X + 3$. What is the MSE (mean squared error) of the output model? *NOTE: Enter your answer to the nearest integer.* [Ans: 4]

Quiz 1 Sep 22

Quiz 2 Sep 22

ET Sep 22

1. Which of the following is true about the function $f(x) = x^2 + y^2 + 2xy$?

   (a) It is Convex function.

   (b) Its Hessian matrix is indefinite.

   (c) (0, 0) is a local minima of this function.

   (d) (0, 0) is a local maxima of this function.

   [Ans: 1, 3]

2. A company manufactures two types of products, A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 products per week and demand for products of type B is at most half of that times production of products of type A. Further, the production level of products of type A can exceed three times the production of products of other type at most 600 units. If the company makes profits of \$12 and \$16 per products respectively on products A and B. If the number of products manufactured of type A and type B are x and y respectively, then formulate this problem so that the company can maximize the profit. Based on the above data answer the given subquestions: The objective function for the problem is

   (a) max $12x + 16y$

   (b) max $50x + 15y$

   (c) min $12x - 16y$

   (d) max 50x - 15y

   [Ans: 1]

3. The constraint function for the problem is

   (a) $x \geq 0$, $y \geq 0$

   (b) $y \geq \frac{x}{2}$

   (c) $x - 3y \leq 600$

   (d) $x + y \geq 1200$

   [Ans: 1, 3]

Quiz 1 Jan 23

Quiz 2 Jan 23

ET Jan 23

1. Which among the following is a convex function?

   (a) $|x| \in \mathbb{R}$

   (b) $e^{-x}, x \in \mathbb{R}$

   (c) $-x^2, x \in \mathbb{R}$

   (d) $\log(1+x), x > -1$

   [Ans: 1, 2]

2. A student wants to purchase a snack from a bakery to meet certain dietary requirements by chossing the best combination of brownies and cheesecake. The student is following some new diet trend which requires her to eat maximum of 6 brownies and maximum of 10 cakes. Also total number of brownies and cakes together the student eats should be greater than 12. The cost of 1 piece of brownie(x_1) and 1 piece of cake (x_2) is 50 units and 80 units respectively. Her goal is to satisfy these requirements at minimal cost.

   (a) In primal linear program, the function to be minimized is $50x_1 + 80x_2$

   (b) Constraints are $x_1 + x_2 > 12, x_1 \leq 6$, and $x_2 \leq 10$

   (c) In primal linear program, the function to be minimized is $6x_1 + 10x_2$

   (d) Constraints are $x_1 + x_2 < 12, x_1 \geq 6$, and $x_2 \geq 10$

   [Ans: 1, 2]

3. A consumer wants to maximize his utility subject to some constraints. He consumes two goods x and y and the utility function is the product of x and y. His budget is Rs.200. The per unit price of goods x and y are Rs.3 and Rs.4 respectively. Choose the correct optimization problem.

   (a) maximize $x + y$ subject to $3x + 4y = 200$

   (b) maximize xy subject to $3x + 4y = 200$

   (c) maximize xy subject to $3x - 4y = 200$

   (d) maximize $x + y$ subject to $3x - 4y = 200$

   [Ans: 2]

Quiz 1 May 23

Quiz 2 May 23

ET May 23

1. Let $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ and $g : \mathbb{R}^d \longrightarrow \mathbb{R}$ be two convex functions. Which of the following functions is/are convex?

   (a) $\max\{f, g\}$

   (b) $\min\{f, g\}$

   (c) $f \circ g$

   (d) $\alpha f + (1 - \alpha)g$ where $\alpha \in \mathbb{R}$

[Ans: 1]

Quiz 1 Sep 23

Quiz 2 Sep 23

ET Sep 23

1. Consider functions $f(x) = \sqrt{x}, x \geq 0$ and $g(x) = |x|, x \in \mathbb{R}$. Then, which among the following options are correct?

   (a) $f \circ g$ is convex.

   (b) $f \circ g$ is not convex.

   (c) $-f$ is convex.

   (d) $h : (0, \infty) \to \mathbb{R}$ defined by $h(x) = f(x) + g(x)$ is convex.

   [Ans: 2, 3]

2. Let $S = \{x_1, x_2, x_3, x_4\} \subseteq \mathbb{R}^d$. Which of the following points must be the part of convex $hull(S)$?

   (a) $0.4x_1 + 0.6x_2$

   (b) $0.4x_1 + 0.2x_2 + 0.2x_3 + 0.2x_4$

   (c) $0.4x_1 - 0.2x_2 + 0.5x_3 + 0.3x_4$

   (d) $0.4x_1 + 0.2x_2 + 0.3x_3 + 0.3x_4$

   [Ans: 1, 2]

Quiz 1 Jan 24

Quiz 2 Jan 24

ET Jan 24

1. Find the maximum value of the function $f(x, y) = xy$ subject to constraints $\frac{x^2}{8} + \frac{y^2}{2} = 2$. [Ans: 4]

2. Consider two functions $f(x, y) = 2x^2 + 3y^2$ and $g(x) = -(x + 1)^2$. Which of the following options is/are true?

   (a) $f(x, y)$ is a convex function.

   (b) $g(x)$ is a convex function.

   (c) $g \circ f$ is not a convex function.

   (d) The local minimum value of the function $f(x, y)$ is the global minimum of the function.

   [Ans: 1, 3, 4]

3. Which of the following following set is/are convex sets in $\mathbb{R}^2$?

   (a) $\{(x, y) | x^2 + y^2 = 1\}$

   (b) $\{(x, y) | x^2 + y^2 \leq 1\}$

   (c) $\{(x, y) | x^2 + y^2 > 1\}$

   (d) $\{(x, y) | \frac{x^2}{4} + \frac{y^2}{9} < 1\}$

   [Ans: 2, 4]

Quiz 1 May 24

Quiz 2 May 24

ET May 24

1. A company produces two types of products P_1 and P_2. The cost price per unit of P_1 and P_2 are 2 and 3, respectively. The production process requires two types of resources: labor hours and machine hours. Each unit of P_1 requires 2 labor hours and 1 machine hour, while each unit of P_2 requires 1 labor hour and 3 machine hours. The company has constraints on the availability of labor and machine hours, which are 80 and 90 hours, respectively. Use the above information to answer the given sub-questions.

   Choose the correct Primal optimization problem from the following.

   (a) Minimize: $3x + 2y$ Subject to: $2x + y \leq 80, x + 3y \leq 90, x \geq 0, y \geq 0$

   (b) Minimize: $2x + 6y$ Subject to: $2x + y \leq 80, x + 3y \leq 90, x \geq 0, y \geq 0$

   (c) Minimize: $3x + 2y$ Subject to: $2x + y \leq 90, x + 3y \leq 80, x \geq 0, y \geq 0$

   (d) Minimize: $2x + 3y$ Subject to: $2x + y \leq 90, x + 3y \leq 80, x \geq 0, y \geq 0$

   [Ans: 4]

2. Find the minimum cost price of the products. [Ans: 80]

3. Consider the following scenarios involving geographical regions (w.r.t some reference of z- axis and y-axis), which of the following regions represents the convex set in $\mathbb{R}^2$.

   (a) A circular park with one side boundary is a circle of radius of 200 meters and another side boundary is a circle of radius of 300 meters, both centered at the origin.

   (b) City district defined by the region above the x-axis within a radius of 10 miles from the origin, forming a semicircle.

   (c) A triangular region with vertices at (0, 0), (1, 0), and (0, 1).

   (d) A path consisting of two connected line segments (forming the boundaries of the path on either sides) formed a "V" shape with vertices at (0, 0), (2, 2), and (2, 0)

   [Ans: 2, 3]

4. Consider the following input data points: Suppose we fit a linear model $f(x) = x_1 +$

y	X
9	(2,3,4)
2	(-1,1,2)
7	(4,2,2)
4	(0,-2,-1)

   $2x_2 + x_3$ where $x = (x_1, x_2, x_3)$. Compute the value of the loss function L for this dataset which is defined as $L = \frac{1}{n}\sum_{i=1}^{n}(f(x^i) - y^i)^2$. Enter the answer correct to one decimal place. [Ans: 4.2]

Quiz 1 Sep 24

Quiz 2 Sep 24

ET Sep 24

1. If S is a convex set and contains the points a and b, which of the following points is/are guaranteed to also belong to S?

   (a) The mid point of a and b.

   (b) The point $2a - b$.

   (c) The point $0.4a + 0.6b$

   (d) The point $a - b$

   [Ans: 1, 3]

2. Which of the following options is/are true?

   (a) $f(x) = x \ln x$ in the interval $[0, \infty]$ is not a convex function.

   (b) $f(x, y) = e^{x+y}$ is a convex function.

   (c) $f(x, y) = -\ln xy$ in the domain $[0, \infty] \times [0, \infty]$ is not a convex function.

   (d) If $f(x) = \ln x$ and $g(x, y) = e^{2x+y}$, then $(f \circ g)(x, y)$ is a convex function.

   [Ans: 2, 4]

3. The mobile manufacturer has developed a profit model that depends on the number of mobiles (x) sold per week, and the number of hours (y) per week in advertising, according to the function $f(x, y) = 48x + 96y - x^2 - 2xy - 9y^2 + 1$. The budgetary constraint function relating to the product cost is given by $20x + 4y = 216$. Which of the following options is/are true, when the company has the maximum weekly profit?

   (a) $x = 10$

   (b) $x = 9$

   (c) The maximum weekly profit is 541.

   (d) The maximum weekly profit is 540.

   [Ans: 1, 3]

Quiz 1 Jan 25

Quiz 2 Jan 25

ET Jan 25

1. Which of the following options is/are true?

   (a) $X = \{(x_1, x_2, ...x_n) | \sum_{i=1}^{n} ax_i = b\}$ is a convex set.

   (b) Any line in $\mathbb{R}^2$ forms a convex set.

   (c) Union of two convex sets is a convex set.

   (d) Any curve in $\mathbb{R}^2$ forms a convex set.

   [Ans: 1, 2]

2. A company manufactures a commodity at two factories. The total cost of manufacturing depends on the quantities, x and y, supplied by each factory. The cost function is $f(x, y) = 2x^2 + xy + y^2 + 500$. The company's objective is to produce 200 units, while minimizing production costs. Which of the following options is/are true?

   (a) The cost function $f(x, y)$ is convex.

   (b) The optimal solution satisfies $x + y = 250$.

   (c) The optimal production quantity from each factory occurs when $x = y$.

   (d) The minimum production cost is achieved when $x = 50$ and $y = 150$.

   [Ans: 1, 4]

3. Find the maximum value of the function $f(x, y, z) = 5x + 7y + z$ on the curve of intersection of the plane $x + y - z = 4$ and the cylinder $x^2 + y^2 = 25$. [Ans: 46]

Quiz 1 May 25

Quiz 2 May 25

ET May 25

1. Consider three functions $f_1(x, y) = x^4+x^2-2xy+y^2$ $f_2(x, y) = x^\alpha y^\beta$, where $x, y, \alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$ $f_3(x) = (x^2 - 3x + 2)(x^2 - 7x + 12)$ Which of the following functions are convex function?

   (a) $f_1(x, y)$ is a convex function.

   (b) $f_2(x, y)$ is a convex function.

   (c) $f_3(x)$ is a convex function in the open interval (2, 3).

   (d) None of these

   [Ans: 1, 3]

Quiz 1 Sep 25

Quiz 2 Sep 25

ET Sep 25

1. Which of the following functions is/are convex on their respective domains?

   (a) $f(x) = e^x$ for $x \in \mathbb{R}$

   (b) $f(x) = x^2$ for $x \in \mathbb{R}$

   (c) $f(x) = \log(x)$ for $x > 0$

   (d) $f(x) = |x|$ for $x \in \mathbb{R}$

   [Ans: 1, 2, 4]

2. A company aims to minimize its production cost, represented by the following quadratic function: $f(x, y) = 3x^2 + 2xy + y^2$

   The company applies the gradient descent algorithm to minimize this function, starting with an initial guess of $(x_0, y_0) = (1, 1)$ and using a step size (learning rate) of $\eta = \frac{1}{t+1}$ where $t = 0, 1, 2, ...$ Based on the above data, answer the given subquestions.

   (a) Find the updated point (x_1, y_1) after the first iteration of the gradient descent algorithm.

      i. (7,3)

      ii. $(-3, -1)$

      iii. $(1, 1)$

      iv. $(4, 2)$

      [Ans: 2]

   (b) Find the value of the objective function $f(x_1, y_1)$ at the updated point. [Ans: 34]